3) $\forall x \in \mathbb{R}^n$, $x = \underset{E_1}{\underset{\cap}{f(x)}} + \underset{E_0}{\underset{\cap}{\underbrace{x - f(x)}}}$ donc $\mathbb{R}^n = E_0 \oplus E_1$

donc $f$ est diagonalisable

$E_0 = \{x \in E \,/\, f(x) = 0x = 0\} = \ker f$

$E_1 = \{x \in E \,/\, f(x) = x\} = \mathrm{Im} f$

EXO 6

$P_A(\lambda) = -\lambda(\lambda-1)(\lambda-4)$

$n = 3$, 3 valeurs propres simples et distinctes donc $A$ diagonalisable

$\lambda_1 = 0 \neq \lambda_2 = 1 \neq \lambda_3 = 4$

$\exists P$ inversible tq $P^{-1}.A.P = D$

$P = ?$

$E_{\lambda=0} = \{v \in \mathbb{R}^3 \,/\, A.V = 0.V = 0\} = [V_1 = (1,-3,1)]$

$E_{\lambda=1} = [V_2 = (-1/2, -2, 1)]$

$E_{\lambda=4} = [V_3 = (1,1,1)]$

$P = \begin{pmatrix} 1 & -1/2 & 1 \\ -3 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ ; $P^{-1} = \begin{pmatrix} 1/2 & -1/4 & -1/4 \\ -2/3 & 0 & 2/3 \\ 1/6 & 1/4 & 7/12 \end{pmatrix}$

$P^{-1}.A.P = D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

2. $P_B(\lambda) = -(\lambda-3)(\lambda-1)(\lambda-2)$

3. $P_C(\lambda) = (\lambda-2)(\lambda-3)(\lambda+2)^2$

$E_{\lambda=-2} = \{v \in \mathbb{R}^4 \,/\, C.V = -2V\}$ ; $C.V = -2V \Leftrightarrow \begin{cases} (1)\; x + 2y + 3t = -2x \\ (2)\; -x + 4y - t = -2y \\ (3)\; 2y - 2z = -2z \\ (4)\; -2t = -2t \end{cases}$

(3) $\Leftrightarrow y = 0$ (2) $\Leftrightarrow x = -t$

$E_{\lambda=-2} = \{(-t, 0, z, t) \,/\, t, z \in \mathbb{R}\} = \{t(-1,0,0,1) + z(0,0,1,0) \,/\, t, z \in \mathbb{R}\}$

$E_{\lambda=-2} = [\{v_1 = (-1,0,0,1), v_2 = (0,0,1,0)\}] \ldots 1$

$\{v_1, v_2\}$ libre .. 2 (exercice)

(1) et (2) $\{v_1, v_2\}$ base de $E_{\lambda=-2}$ donc $\dim E_{\lambda=-2} = 2 = \text{multipli}_{\lambda=-2}$

C diagonalisable